DISPLACEMENT-BASED DESIGN OF RC BRIDGE COLUMNS IN SEISMIC REGIONS

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SUMMARY

This paper presents a seismic design philosophy based on displacements rather than forces. By inverting the seismic design process, a rational method is established where member strength and stiffness depend on the target displacement. A comprehensive procedure for displacement-based design of cantilever bridge columns is presented and verified by dynamic inelastic time history analysis. Parameter studies are used to examine the influence of several variables within the possible design solution space.

1. INTRODUCTION

True displacement-based design is a seismic design methodology that uses displacements as the basis for the design procedure. In the displacement-based design procedure, the engineer performs seismic design by specifying a target displacement rather than a displacement limit. Strength and stiffness are not variables in the procedure - they are the end results. This paper presents a displacement-based design procedure that is applied to the design of single bridge columns as well as some analysis results that show the validity of the procedure. Also examined is the sensitivity of design results to variations in column size with respect to reinforcement content and ductility demand. It is shown that the design process may be developed in a way that provides the designer with a range of options which all satisfy the target displacement (within the accuracy of the analysis). In a companion paper 1, the procedure is extended to the design of multi-degree of freedom bridges with flexible superstructures.

The fundamental basis of seismic design is still the assumption that an elastic (or modified elastic) acceleration response spectrum provides the best means for establishing required performance of a structure. The limitations of the approach are well known and accepted because of the design convenience, and because of the lack of a viable design alternative. The following summarizes the limitations of the current approach:
(1) Response is based on a 'snapshot' of structural response: that is, response at the moment of peak base shear for an equivalent elastically responding structure. Duration effects, which tend to be period-dependent, are not considered even though they influence the behavior of short-period structures as they suffer a greater number of response cycles than long-period structures. The merits of using modal combination rules to provide some insight into the higher mode effects seems hardly worth while when these will have to be considered by largely empirical rules later in the design process, if protection against undesirable modes of inelastic deformation is made in accordance with capacity design principles.

(2) The relationship between peak displacement response of elastic and inelastic systems is complex, and more variable than commonly accepted. Various rules, such as the “equal energy” and “equal displacement” rules are commonly employed, but without much consistency or logic. Consider a typical elastic acceleration response spectrum as in Figure 1a. There are four distinct zones which can be identified. At zero period, the structure will be subjected to peak ground acceleration (PGA), regardless of ductility capacity, and will fail if a lesser strength than that corresponding to PGA is provided. In the rising portion of the acceleration spectrum, displacements of inelastic systems are usually greater than those of elastic systems with equivalent initial stiffness, and the “equal energy” relationship has some application. In the initial stages of the falling portion of the acceleration spectrum, elastic and inelastic displacement responses are often similar, leading to the “equal displacement” rule. As the structural flexibility increases still further, the “equal displacement” rule tends to become increasingly conservative. At very long periods, there is essentially no structural response to the ground motion since the inertia mass remains stationary in the absolute coordinate reference system. The concept of a constant displacement (independent of period or ductility) could be advanced, where the relative displacement of the center of mass of the structure is equal to the absolute peak ground displacement. Figure 1b represents a typical
design displacement response spectra for damping ratios of 2%, 5% and 10%. Note the constant displacement region for large periods.

Although these points have been recognized and partially considered in codes which define inelastic spectra with variable ratios between elastic and ductile coordinates, such as the New Zealand Loadings Code\textsuperscript{3}, confusion is still widespread.

(3) It is apparent from the comments in (2) above that the elastic acceleration approach places excessive emphasis on initial elastic stiffness characteristics of the structure and its elements. Despite this emphasis, designers are generally less careful than they should be in determining these characteristics. Among the problems encountered are:

(1) The elastic stiffness of reinforced concrete members is frequently taken to be that of the gross uncracked section, which is inappropriate for response at levels close to yield.

(2) Influences of foundation flexibility on elastic properties are frequently ignored. The question remains as to whether alternative and better design philosophies might be considered. It is the thesis of this paper, and the companion paper\textsuperscript{1}, that more consistent results may be achieved by reversal of the design process, starting with displacements, and ending with elastic characteristics.

2. THE SUBSTITUTE STRUCTURE APPROACH

The displacement-based design procedure presented in this paper requires the use of the \textit{Substitute Structure Approach}\textsuperscript{5,6}. The substitute structure approach is a procedure where an inelastic system is modeled as an equivalent elastic system. The equivalent elastic system is known as the substitute structure and has properties of 1) Effective Stiffness, $K_{\text{eff}}$, 2) Effective Damping, $\zeta$ and 3) Effective Period, $T_{\text{eff}}$. Figure 2 represents a bilinear approximation to the structural force-displacement response of a single degree of freedom system. The stiffness of the cracked section, $K_{\text{cr}}$, is based on a cracked section analysis at first yield of the flexural reinforcement. A post-yield stiffness, $K_{\text{eo}}$, is based on results of a moment curvature analysis.
The effective stiffness, $K_{\text{eff}}$, is the secant stiffness to maximum displacement, $\Delta_u$. The effective damping, $\zeta$, is related to the hysteretic energy absorbed. Since the effective properties of the substitute structure are elastic, a set of elastic response spectra can be used for design. Therefore, the substitute structure approach allows an inelastic system to be designed and analyzed using elastic response spectra.

3. DISPLACEMENT-BASED DESIGN OVERVIEW

The basic philosophy behind displacement-based design as used in this approach is that a structure in a seismic region can be designed based on a specified displacement. The specified displacement may be characterized by either serviceability or ultimate criteria. In both cases, the criteria are likely to be defined by strain limits which can be related to displacements or drift by structure geometry.

For a reinforced concrete bridge column, the serviceability limit state might be taken to be the onset of cover concrete crushing, or the development of crack widths of a size that might require injection grouting after an earthquake. The first criterion can be related to extreme compression fiber concrete strain, while the second relates to strain in the tension reinforcing bars at maximum distance from the neutral axis.

“Ultimate” conditions may be taken as corresponding to a “damage control” limit state beyond which structural repair is not economically feasible, or alternatively a true “collapse” limit state. Normally, the former is adopted in design. A maximum extreme fiber compression strain can be set based on the volumetric ratio of transverse reinforcement in the plastic hinge region. Reinforcement strain limits will be related to provision of adequate protection against bar buckling or low cycle fatigue. Both limit states, as discussed above, imply a degree of inelastic response. It is comparatively straightforward to relate these limit states to acceptable plastic curvatures, and thus to plastic drift ratios. Total drift (elastic plus plastic) can then be established and from the
structure geometry, the specified displacement limits defined. The approach outlined in
the following can be carried out based on either specified total or plastic drift.

In order to design for a displacement, a set of displacement response spectra
(DRS) must be employed. Figure 3 represents DRS for various damping ratios generated
from an artificial accelerogram designed to satisfy the EuroCode\(^4\) design acceleration
response spectrum scaled to a peak ground acceleration of 0.6g. Given values for the
specified displacement and effective damping, the elastic displacement response spectra
can be used to obtain a value for the effective period of the substitute structure, \(T_{\text{eff}}\). The
effective stiffness, \(K_{\text{eff}}\) can then be calculated and the structure designed. The following
section provides a step by step procedure illustrating displacement-based design.

4. DISPLACEMENT-BASED DESIGN PROCEDURE

The following procedure has been developed for isolated bridge columns\(^7\),
although it will be shown in the companion paper\(^1\) that it can be modified for the design
of entire bridge structures. A design example is provided in the appendix.

**Step 1: Choose Initial Parameters**

**Step 1a: Establish Column Axial Load and Column Height**

Lumped mass at top of column = \(M\)

Height of column = \(L\)

**Step 1b: Choose Material Properties**

\(f'_c = \text{Concrete Compressive Strength}\)

\(f_y = \text{Longitudinal Reinforcement Yield Strength}\)

\(E = \text{Young's Modulus of Concrete}\)

**Step 1c: Establish the Target Displacement, \(\Delta_u\)**

The value for the target displacement depends on the design limit
state, as discussed previously.
Step 1d: Choose an Effective Damping Relationship

The damping relationship is derived considering the effect of ductility on damping and is related to the hysteretic energy absorbed. The relation shown in Figure 4a is based on the Takeda hysteretic model, Figure 4b, for an unloading stiffness factor of $n = 0.5$ and a bilinear stiffness ratio of $r = 0.05$. It also includes an additive term of 5% viscous damping. The relation is given as Eq. 1.

$$\zeta = 0.05 + \left(1 - \frac{0.95}{\sqrt{\mu}} - 0.05/\mu\right) \frac{1}{\pi}$$

(1)

Step 1e: Choose the Displacement Response Spectra

The displacement response spectra from Figure 3 can be smoothed or linearized as in Figure 5.

Step 2: Calculate Parameters Leading to Effective Stiffness

Step 2a: Select the starting value for the yield displacement, $\Delta_y$

The following value for the yield displacement is suggested as an initial guess, though the solution is insensitive to this value.

$$\Delta_y = 0.005L$$

(2)

Step 2b: Calculate the initial displacement ductility, $\mu$

$$\mu = \frac{\Delta_u}{\Delta_y}$$

(3)

Step 2c: Determine the Effective Damping, $\zeta$

Enter the damping curve with the value for ductility found in Step 2b and obtain the equivalent viscous damping ratio, $\zeta$ (see Figure 4 and Eq. 1).

Step 2d: Determine the Effective Period, $T_{eff}$

Enter the displacement response spectra with the value for $\Delta_u$, read across to the intersection with the appropriate response curve (given by the value of damping from Step 2c), and read down to find the effective period, $T_{eff}$ (see Figure 5).
Step 2e: Determine the Effective Stiffness, $K_{eff}$ (See Figure 2)

From consideration of a single degree of freedom oscillator,

$$K_{eff} = \frac{4 \pi^2 M}{T_{eff}^2}$$  \hspace{1cm} (4)

Step 3: Obtain Design Forces

Step 3a: Determine the Ultimate Force, $H_u$, and Ultimate Moment, $M_u$

Since the substitute structure is elastic, and again referring to Figure 2,

$$H_u = K_{eff} \times \Delta_u$$  \hspace{1cm} (5)

$$M_u = H_u \times L$$  \hspace{1cm} (6)

Step 3b: Determine the Design Force, $H_d$, and Moment, $M_d$

Based on a bi-linear force-displacement model ($r$ is defined as the stiffness ratio $K_{eo}/K_{cr}$), the initial estimate of ductility, $\mu$, and the ultimate force, $H_u$, a first estimate for the design force is obtained using Equation (7) 7.

$$H_d = \frac{H_u}{r\mu - r + 1}$$  \hspace{1cm} (7)

$$M_d = H_d \times L$$  \hspace{1cm} (8)

Step 4: Design the Column

Step 4a: Estimate an Initial Column Diameter

Based on experience and the value for the design moment obtained in Step 3b, estimate an initial diameter for the column. In practice, a range of possibilities may be considered.

Step 4b: Design the Column Rebar

Design the column according to the chosen diameter, the estimate for the design moment, $M_d$, and the axial load, $P$. A steel ratio in the range of $0.70% < \rho_l < 4\%$ is suggested. If the design yields a value for $\rho_l$ outside this range, a different column diameter should be considered. Increasing the steel above the design value may decrease the displacement, but it is important to note that if this is done, the shear and confinement reinforcement must be
based on the moment capacity of the revised section in accordance with capacity design principles.  

**Step 4c: Estimate Second Moment of Area of the Cracked Section**

Figure 6 represents a relation for the effective second moment of area of the cracked section that includes the effect of longitudinal steel ratio and axial load ratio. Figure 6 can be expressed as Equation (9):

\[
\frac{I_{cr}}{I_g} = 0.21 + 12\rho_t + \left(0.1 + 205(0.05 - \rho_t)^2\right) \times \frac{P}{f'_c A_g} \tag{9}
\]

Where

- \(I_{cr}\) is the second moment of area of the cracked section at first yield.
- \(I_g\) is the second moment of area of the gross section.
- \(A_g\) is the column gross section area.

**Step 4d: Calculate Column Elastic Stiffness**

For a single degree of freedom cantilever, the stiffness is

\[
K_{cr} = \frac{3EI_{cr}}{L^3} \tag{10}
\]

**Step 5: Optional Steps**

The procedure will work if these steps are omitted, but the end result may be a column with either very high steel ratio, or no steel at all. Therefore, it is useful as a guideline to choosing column diameter.

**Step 5a: Calculate the Column Period Corresponding to elastic stiffness, \(T_{cr}\)**

\[
T_{cr} = \sqrt{\frac{4\pi^2 M}{K_{cr}}} \tag{11}
\]

**Step 5b: Calculate the Post-Yield Stiffness, \(K_{eo}\), and Corresponding Period, \(T_{eo}\)**

\[
K_{eo} = r \times K_{cr} \tag{12}
\]

\[
T_{eo} = \sqrt{\frac{4\pi^2 M}{K_{eo}}} \tag{13}
\]
Step 5c: Determine Problem Status

By checking where the value for the effective period lies in relation to $T_{cr}$ and $T_{eo}$, we can determine if we should proceed further in the process, or revise the design.

1. If the effective period, $T_{eff}$, is bounded but not close to $T_{cr}$ and $T_{eo}$, then proceed to Step 6a.

2. If the effective period, $T_{eff}$, is bounded but very close to the period of the cracked section, $T_{cr}$, then proceeding with displacement-based design will yield a column with very high steel ratio (typically greater than 4%) and low ductility demand. A better solution would be to increase the column diameter and return to Step 4b. Figure 7a illustrates this scenario in terms of the effective stiffness.

3. If the effective period, $T_{eff}$, is bounded but very close to the post-yield period, $T_{eo}$, then any strength can be provided and the column will be adequate. Proceeding with displacement-based design will yield a column with very low reinforcement and very high ductility demand. A better solution is to either decrease the column diameter or use the nominal minimum longitudinal reinforcement ratio of 0.70%. Capacity design principles must be followed, as the strength of the column changes. Figure 7b illustrates this scenario in terms of the effective stiffness.

Step 6: Obtain Revised Yield Displacement and Check Convergence

Step 6a: Calculate the Yield Displacement, $\Delta_y$

Based on the stiffness from Step 4d,

$$\Delta_y = \frac{H_d}{K_{cr}} \quad (14)$$

Step 6b: Check Convergence

If the difference between $\Delta_y$ from Step 6a and $\Delta_y$ from Step 2a is greater than a specified tolerance (5% is suggested), then return to Step 2b
with the $\Delta_y$ from Step 6a to obtain a revised displacement ductility, and hence revised damping estimate. The procedure cycles between Steps 2b through 6a until convergence.

**Step 7: Perform Design for Transverse Reinforcement**

Transverse reinforcement must be designed to satisfy requirements for confinement and shear strength. Confinement requirements are obtained from the required displacement ductility as follows:

The curvature ductility demand is obtained from the required displacement ductility demand by the following relationship

$$
\mu_\phi = 1 + \left( \frac{\mu_\Delta - 1}{3} \right) \left( \frac{L_p}{L} \right) \left[ 1 - 0.5 \left( \frac{L_p}{L} \right) \right]
$$

(15)

Hence, the required ultimate curvature, $\phi_u$, is found from

$$
\phi_u = \mu_\phi \phi_y
$$

(16)

where the yield curvature is related to the yield displacement by the relationship

$$
\phi_y = \frac{3\Delta_y}{L^2}
$$

(17)

The required extreme fiber compression strain, $\varepsilon_{cu}$, at maximum response is then

$$
\varepsilon_{cu} = \phi_u \varepsilon_u
$$

(18)

Where $\varepsilon_u$ is the neutral axis depth at maximum response, typically found from a moment-curvature analysis. Finally, the relationship between volumetric steel ratio of transverse reinforcement and ultimate compression strain is based on the 'energy balance' approach of Mander et. al. $^{12}$ as simplified by Chai et. al. $^{13}$, namely

$$
\varepsilon_{cu} = 0.004 + \frac{1.4\rho_s f_{yh} \varepsilon_{sm}}{f'_{cc}}
$$

(19)

is manipulated to yield the required value of $\rho_s$ as
In Eqs. 19 and 20, \( f'_{cc} \) is the confined strength of the concrete, which may be established from the approach of Mander et. al., and \( f_{yh} \) and \( \varepsilon_{sm} \) are the yield strength, and strain at ultimate strength of the transverse reinforcement. For this study, U.S. grade 60 reinforcement (\( f_y = 414 \) MPa nominal) was assumed, with probable values of \( f_y = 455 \) MPa and \( \varepsilon_{sm} = 0.10 \) adopted for the analysis.

Shear reinforcement was based on the recently developed model of Priestley et. al. \(^{11}\), which defines the shear strength of a member by a three component additive model of the form:

\[
V_n = V_c + V_s + V_p
\]

where \( V_c, V_s, \) and \( V_p \) are components of shear strength resulting from concrete shear resisting mechanisms, truss mechanisms involving transverse reinforcement, and an axial force component in the form of a diagonal compression strut of magnitude equal to the axial force.

Separation of the axial force component from the concrete component significantly improves the ability of the model to predict shear strength when compared with other models \(^{11}\). However, for the range of parameters considered in this study, design of transverse reinforcement for confinement was more critical than for shear strength, so the individual components of Equation 21 are not presented herein. The interested reader is referred to Priestley, et. al. \(^{11}\) for more details.

5. TYPICAL DISPLACEMENT-BASED DESIGN RESULTS

The procedure outlined in section 4 is suitable for inclusion in an automated design procedure. Simplified relationships between displacement, damping, and period shown in Figure 5 are digitized, with interpolation during the design process. For this
study, a rigid-based simple cantilever, as shown in Figure 2a, of height in the range of 5-15m supporting an inertia weight of 4905 kN was adopted. Other variables included the specified drift, and the option of total or plastic drift usage.

By performing displacement-based design, a range of possible solutions that depend on the column diameter chosen can be obtained. Figures 8 through 11 illustrate design results for columns designed for an ultimate drift ratio of 3% and axial load of 4905 kN. An ultimate drift level of 3% is typical of levels expected from current design, but conservative when compared with displacement capacities of well confined columns, which typically indicate that an ultimate drift ratio of 5% can be dependably achieved 14,15.

Figure 8 indicates that for a given column height, the design moment decreases as the diameter increases. This might be considered counter intuitive, but occurs as a result of the higher ductility and hence larger force-reduction factor possible with larger and hence stiffer columns. A consequence of this trend is that the longitudinal steel ratio also decreases as the column diameter increases, and does so at a faster rate than the design moment reduction, as shown in Figure 9. Also plotted in Figure 9 are the practical limits of \(0.007 \leq \rho_l \leq 0.04\). It is noted that the range of possible column diameters is somewhat restricted.

As expected, displacement ductility demand, shown in Figure 10, increases as the column diameter increases, and as the column height decreases. All values of ductility for column heights in the range of \(8 \leq L \leq 15\)m are within commonly accepted limits. For \(L=5\)m, ductility levels are high for the larger diameters, but ductility capacity is also typically high for such columns because of the lower aspect ratio and low longitudinal steel ratio.

An interesting consequence of the design process, illustrated in Figure 11, is that the volumetric ratio of transverse reinforcement for the plastic hinge region is relatively insensitive to column diameter, and depends almost entirely on column height. This trend
is especially apparent for column heights 8m and above. In these analysis, it was found that confinement requirements, rather than shear governed in all cases. As a consequence of this insensitivity of \( \rho_s \) to \( D \), decisions on optimum diameter would not need to include consideration of \( \rho_s \). It is suggested that a minimum value of \( \rho_s = 0.4\% \) be adopted to prevent longitudinal bar buckling \(^{18}\). From Figure 11 it is apparent that columns 10m and taller are governed by this requirement.

Several designs were also performed by specifying a plastic drift ratio. Figures 12 through 15 illustrate design results for columns designed for a plastic drift ratio of 3% and axial load of 4905 kN. The choice of a limitation on plastic drift is more realistic than one on total drift since yield drift is highly dependent on column aspect ratio.

Figure 12 presents the variation in design moment as the column diameter is changed for a specified column height. The same trends are observed as in the columns designed for a total drift ratio of 3%. Figure 13 illustrates that the variation in the required longitudinal steel also follows the same trends as before, but with reduced spread.

The variation of displacement ductility demand versus diameter for various column lengths is plotted in Figure 14, and the required transverse volumetric steel ratio is plotted in Figure 15. Note that in Figure 15 there is a trend for a decrease in required steel ratio with increasing column diameter, despite the increased displacement ductility demand. Also, only the 12m tall, 1.5m diameter column is governed by the minimum \( \rho_s \) requirement of 0.4%.

6. VERIFICATION AND DISCUSSION OF DESIGN APPROACH

Several designs were performed at various target displacements, represented by either target total drifts or plastic drifts. The resulting designs were then modeled as inelastic single degree of freedom cantilevers with hysteretic characteristics defined by the Takeda degrading stiffness model represented in Figure 4. These models were then subjected to the artificial accelerogram representing the EuroCode 8 design spectra of
Figure 5. The results of the analysis are presented in Figures 16 through 18 where the drifts shown are those from the dynamic inelastic time history analysis and represent peak displacement response. Also shown in each graph is the target drift used in the designs. Generally, the columns sustained peak displacements within 20% of the design displacements, although occasional larger discrepancies were noted, particularly for the shorter (and hence shorter period) structures, where results from the time history analysis tended to be significantly less than the target values. The main reason for these discrepancies appears to be the linearization of the displacement spectra adopted for design (Figure 5), which results in significant departure from the computed displacement spectra. This is particularly apparent in the range of 1 to 3 seconds, for effective damping ratios of 10 to 20% which covers the range of effective periods of the shorter bridge columns, and where computed spectral displacements are up to 30% less than the linearization adopted for design. In view of these approximations, the agreement is felt to be acceptable.

The intent of this paper has been to demonstrate that design of simple bridge systems can be based on specified displacements. A further advantage of this approach is that it enables a quick assessment of sensitivity to $P$-$\Delta$ influences. Although there is still considerable controversy over the way in which $P$-$\Delta$ effects should be included in design, dynamic inelastic analysis of a wide range of bridge piers under different accelerograms\[16\] indicate that there is no consistent increase in displacements resulting from $P$-$\Delta$ effects provided that the strength to weight ratio satisfies Eq. 22:

$$\frac{H_d}{W} \geq k \frac{\Delta_u}{L}$$

(22)

where $k = 3.3$. For design purposes, extra conservatism is appropriate, and a value of 4 has been suggested\[17\]. With $W = 4905kN$, and a drift limit of 3%, Eq. 22 requires $M_d > 589L$. Examination of Figure 8 indicates that all designs represented in that figure satisfy this requirement, although the values for $L = 12m$ and $L = 15m$ for $D = 1.65m$ are
marginal. It is again of interest to note that the larger diameter columns are most critical according to the criteria of Eq. 22. From Figure 12, we note that several of the designs would not satisfy the $P-\Delta$ requirement and that the design of these columns would require consideration of the $P-\Delta$ effect.

A concern with displacement-based design would be the potential errors in the displacement spectra as a result of the double integration from the acceleration record. This might be particularly the case for very long period structures, where spectra can be influenced by baseline corrections. Although it is certainly desirable that the precision of displacement spectra be as high as possible, it must be recognized that the same concerns apply equally to force based design. If we accept that it is the peak response displacement of a structure designed to force based criteria that is of ultimate concern, the same degree of uncertainty exists in the ability to predict this as exists in the displacement spectra used for displacement-based design. The uncertainty will in fact be larger for force based design systems because of the coarseness of the “equal displacement” and “equal energy” approximations typically used to estimate peak inelastic displacements from elastic response levels. It is felt that a significant advantage that displacement-based design provides the engineer is the elimination of the various approximations that are applied to elastic acceleration response spectra in force based design.

7. CONCLUSIONS

A procedure for displacement-based design was presented. This procedure addresses the following problems with the force based approach: (1) Eliminates the need for the use of a force reduction factor. (2) Addresses service and ultimate limit states using the same design procedure. (3) Provides a rational seismic design procedure that is compatible with the philosophy that structures are designed to undergo plastic deformation in a large earthquake while satisfying service criteria in smaller earthquakes.
The procedure presented is a “pure” form of displacement-based design where the only initial design parameters are the column height and target displacement. Strength, stiffness and reinforcement details are a result of the design procedure and are dependent on the target displacement chosen.

The procedure developed in this paper has considered only simple single degree of freedom systems. The companion paper considers more complex systems where peak response can still be adequately characterized by a fundamental mode. Further work is needed to extend the approach to designs where a number of significant modes of response must be considered. It would appear that elastic modal analysis based on properties of the substitute structure should provide a means for examination of the more complex systems.

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APPENDIX - DESIGN EXAMPLE FOR A SINGLE BRIDGE COLUMN

**Step 1a:**
Lumped mass at top of column = 500,000 kg
Height of column = 5 m

**Step 1b:**

\[ f'_c = 40 \text{ MPa} \]
\[ f_y = 400 \text{ MPa} \]
\[ E = 31.62 \text{ GPa} \]
The goal is to determine a suitable stiffness and the corresponding reinforcement using displacement-based design.

**Step 1c:**

Step 1c is to establish an acceptable target displacement. This value comes from a combination of design constraints such as serviceability, and a limit on displacement such that instability or excessive deformation not occur. An ultimate drift ratio of 3% is chosen.

Therefore,

\[
\text{Target Displacement} = 0.03 \times 5\text{m} = 0.150\text{ m}
\]

**Steps 1d, 1e:**

Steps 1d and 1e are to choose a damping relationship and a displacement response spectra. Figure 3 and Figure 4 represent the damping relation and DRS used for this example.

**Step 2a:**

Step 2a is to determine an initial guess for the yield displacement. Consider the following

\[
\Delta_y = 0.005 \times L = 0.025\text{ m}
\]

**Step 2b:**

Step 2b is to determine the initial ductility

\[
\mu = \frac{\Delta_u}{\Delta_y}
\]  
(Eq. A1)

Therefore,

\[
\mu = \frac{0.150}{0.025} = 6.0
\]

**Step 2c:**

In Step 2c, the value of effective damping that corresponds to a ductility of 6.0 from Step 2b is obtained. Using Figure 3, \( \zeta = 20.6\% \)
**Step 2d:**

Enter the displacement response spectra with the value for target displacement, $\Delta_u$, and read across to the appropriate response curve. Read down to find a value for the effective period, $T_{eff}$, and obtain $T_{eff} = 1.627$ sec.

**Step 2e:**

Step 2e is to find the value for the effective stiffness, $K_{eff}$, based on the effective period, $T_{eff}$. This is given by Equation A2 below.

$$K_{eff} = \frac{4\pi^2 M}{T_{eff}^2}$$  \hspace{1cm} (Eq. A2)

$$K_{eff} = \frac{4 \times \pi^2 \times 500,000}{1.627^2} = 7.452 \text{ MN/m}$$

**Step 3a:**

Solving for the ultimate force and ultimate moment from Equations A3 and A4:

$$H_u = K_{eff} \times \Delta_u$$  \hspace{1cm} (Eq. A3)

$$M_u = H_u \times L$$  \hspace{1cm} (Eq. A4)

Therefore,

$$H_u = 7,452,000 \times 0.15 = 1118 \text{ kN}$$

and

$$M_u = 1118 \times 5 = 5589 \text{ kN} \cdot \text{m}$$

**Step 3b:**

In Step 3b determine a value for the design force and design moment based on Equations A5 and A6.

$$H_d = \frac{H_u}{r\mu - r + 1}$$  \hspace{1cm} (Eq. A5)

$$M_d = H_d \times L$$  \hspace{1cm} (Eq. A6)

Therefore,

$$H_d = \frac{1,118,000}{0.05 \times 6.0 - 0.05 + 1} = 894.3 \text{ kN}$$

and
\[ M_d = 894.3 \times 5.0 = 4471 \text{ kN.m} \]

**Step 4a:**

In Step 4a, the engineer chooses a column size based on experience. The process will either tell the engineer the required strength, or that the column size must be changed. Consider a column with the following diameter.

\[ D = 1.1 \text{ m} \]

**Step 4b:**

In Step 4b, the design of the column reinforcement for a design moment of 4471 kN·m with a column diameter of 1.1 meters is performed. Several commercial programs are available to design circular columns. Alternatively, column design charts could be used for the designs as well. A program developed by King \(^{10}\) was used in this paper. The result of the design is shown below in terms of the required steel ratio.

\[ \rho_l = 1.76 \% \]

**Steps 4c, 4d:**

In Step 4c and 4d, solve for the second moment of area and stiffness of the cracked section. Equation (A7) below gives the stiffness as a function of the second moment of area of the cracked section, which is given in Equation A8.

\[
K_{cr} = \frac{3EI_{cr}}{L^3} \quad \text{(Eq. A7)}
\]

\[
\frac{I_{cr}}{I_{gr}} = 0.21 + 12\rho_l + (0.1 + 205(0.05 - \rho_l)^2) \times \frac{P}{f'c A_g} \quad \text{(Eq. A8)}
\]

The second moment of area for a circular column is given by Equation A9.

\[
I_g = \frac{\pi \times D^4}{64} \quad \text{(Eq. A9)}
\]

Therefore, for \( D = 1.1 \text{ meters} \),
\[ I_g = \frac{\pi \times 1.1^4}{64} = 0.072 \text{ m}^4 \]
\[ I_{cr} = 0.462 \times 0.072 = 0.033 \text{ m}^4 \]

Solving for \( K_{cr} \),
\[ K_{cr} = \frac{3 \times 31.62 E9 \times 0.033}{5^3} = 25.19 \text{ MN/m} \]

**Step 5a:**

Steps 5a through 5c were classified as optional in the procedures. For the purpose of this example, these steps will be shown so that a physical meaning can be attached to them.

Step 5a in the procedure is to calculate the period associated with the stiffness of the cracked section.

\[ T_{cr} = \frac{4\pi^2 M}{K_{cr}} \]  
(Eq. A10)

Therefore,
\[ T_{cr} = \frac{4\pi^2 \times 500,000}{25,190,000} = 0.885 \text{ sec.} \]

**Step 5b:**

Step 5b is to find the minimum stiffness and minimum period associated with the plastic behavior of the structure, known as the post-yield stiffness and post-yield period (Equations A11 and A12). For this example, a stiffness ratio of \( r = 0.05 \) has been adopted.

\[ K_{eo} = r \times K_{cr} \]  
(Eq. A11)

\[ K_{eo} = 0.05 \times 25,190,000 = 1.26 \text{ MN/m} \]

\[ T_{eo} = \sqrt{\frac{4\pi^2 M}{K_{eo}}} \]  
(Eq. A12)

\[ T_{eo} = \sqrt{\frac{4\pi^2 \times 500,000}{1,259,000}} = 3.959 \text{ sec} \]
Step 5c:

Because the effective period of the substitute structure is bounded by $T_{cr}$ and $T_{eo}$ ($0.885 < 1.627 < 3.958$), continue on to the next step. If the effective period were very close to either of the limits, the procedure would proceed in a different manner. It was stated earlier that it is not necessary to perform this check. If this check were not performed and the values for the period fell near the limits of the prescribed range, it would have manifested itself in the form of very large steel content or no steel at all.

Step 6a:

Solving for the revised yield displacement (Eq. A13),

$$\Delta y_{n+1} = \frac{H_{dn}}{K_{crn}}$$

(Eq. A13)

$$\Delta y_{n+1} = \frac{894,300}{25,190,000} = 0.035 \text{ m}$$

Step 6b:

Compare $\Delta y_{n+1}$ with $\Delta y_n$, to determine if the revised yield displacement has converged to the previous guess for the yield displacement.

$\Delta y_n = 0.025$ meters, and $\Delta y_{n+1} = 0.035$ meters, which is a difference of 28.6%.

Second Iteration of Procedure:

Therefore, return to Step 2b and re-iterate replacing $\Delta y_n$ with $\Delta y_{n+1}$.

The calculations for the second iteration are as follows:

$$\mu = \frac{0.150}{0.035} = 4.29$$

From the damping curve,

$$\zeta = 18.9 \%$$

From the displacement response spectra,

$T_{effn+1} = 1.55 \text{ sec}$

and

$K_{effn+1} = 8.24 \text{ MN/m}$

Therefore,
\[ H_{u_{n+1}} = 1235.25 \text{ kN} \]

and

\[ H_{d_{n+1}} = 1061 \text{ kN} \]

\[ M_{d_{n+1}} = 5304 \text{ N} \cdot \text{m} \]

Redesign the column for the above design moment to obtain the revised steel ratio.

\[ \rho = 2.41 \% \]

Now, find the second moment of area of the cracked section associated with this amount of steel,

\[ I_{cr} = 0.038 \text{ m}^4 \]

Solving for the revised column stiffness,

\[ K_{cr} = 28.9 \text{ MN/m} \]

Solving for the revised yield displacement,

\[ \Delta_y = 0.037 \text{ m} \]

Now, the difference between the updated \( \Delta_y \) and the previous \( \Delta_y \) is

\[ \frac{0.037 - 0.035}{0.037} = 5.4\% \]

In just two iterations, the result has converged within 5%. This result could be further improved, but for the purpose of this example it is terminated here.

REFERENCES


Figure 2  Substitute Structure Approach for Seismic Response of a Bridge Pier
Figure 7 Implications of Effective Stiffness Near the Extremes of the Solution Range
Figure 4b  Takeda Degrading Stiffness Hysteresis