

RC-Analysis

Moment-Curvature and Shear Strength Analysis of Reinforced Concrete Sections

Prepared by:

Vinicio A. Suarez and Jose C. Hurtado



RC-Analysis is a simulation tool of the:

Virtual Laboratory for Earthquake Engineering

www.utpl.edu.ec/vlee

Universidad Técnica Particular de Loja

Ecuador, 2008

Disclaimer

There are no warranties, explicit or implicit, that this program is free of error, or is consistent with any particular standard of accuracy, or that it will meet your requirements for any particular application. It should not be relied on for any purpose where incorrect results could result in loss of property or personal injury. If you do use the program for any such purpose it is at your own risk. The authors disclaim all liability of any kind, direct or consequential, resulting from the use of this program.

If you find that something does not seem to work correctly with RC-Analysis, please report the problem to vasuarez@utpl.edu.ec. This program is available free of charge as part of the Virtual Laboratory for Earthquake Engineering VLEE.

INTRODUCTION

Most of buildings and bridges are designed to exceed their elastic limits when attacked by the design earthquake. Exceeding elastic limits for reinforced concrete sections means cracking of concrete, yielding of reinforcement, crushing of concrete and eventual collapse of the section.

The nonlinear flexural behavior of reinforced concrete sections can be assessed by a special type of section analysis called Moment-Curvature analysis. The outcome of this analysis is the relation between applied moment and related curvature in the section. Other very important information is the relation between curvature and concrete strain, steel strain and neutral axis depth.

The theory of beam flexure states that curvature is the second derivate of displacement. Moment-curvature relation is then very important since gives the correspondence between moment and curvature necessary to the calculation of displacement due to forces acting in inelastic elements. Furthermore, it allows the assessment of displacement at different limit states of damage.

PROGRAM DESCRIPTION

RC-Analysis produces a Moment-Curvature curve coupled to a shear strength curve for different types of concrete sections. The program is available free of charge to registered users of the Virtual Laboratory of Earthquake Engineering, and can be accessed on-line from www.utpl.edu.ec/vlee.

The program is operated on-line. No downloads or installation is required. The programs features a web interface (Fig. 1) where the user inputs the section geometry, reinforcing and material properties. The analysis is performed by clicking the "Run Analysis" button. The output is presented in two sections, one containing the response plots along with the most important analysis values and another including a text file with all the analysis information.

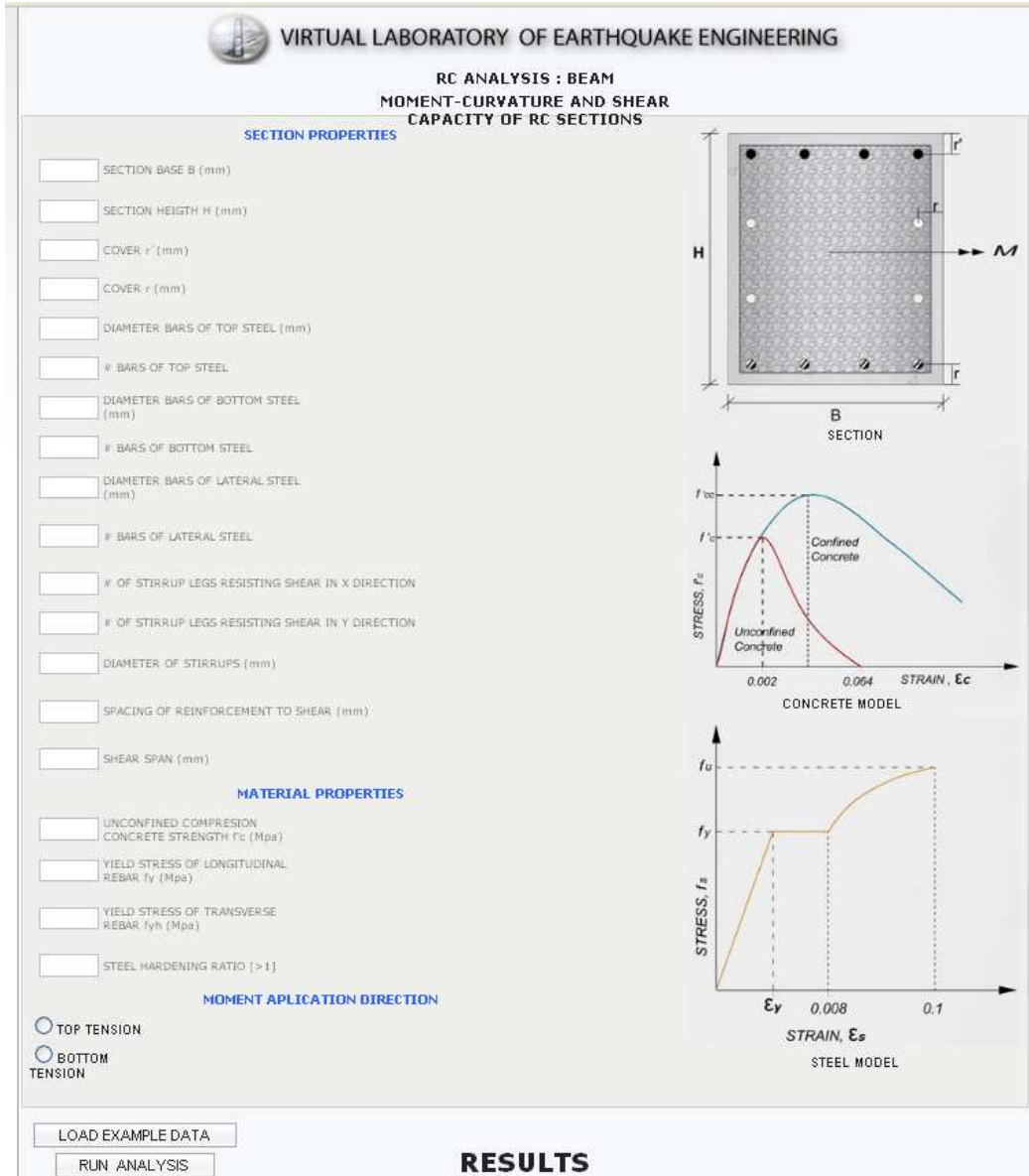


Figure 1 RC-Analysis Interface

Currently RC-Analysis supports beam sections (Fig. 2), rectangular column sections and circular column sections. For beam analysis the user can choose to put the top or bottom of the section in tension. In the rectangular column analysis the user can specify an angle for application of moment. This angle should correspond to the angle of the moment resultant in the case of biaxial loading.

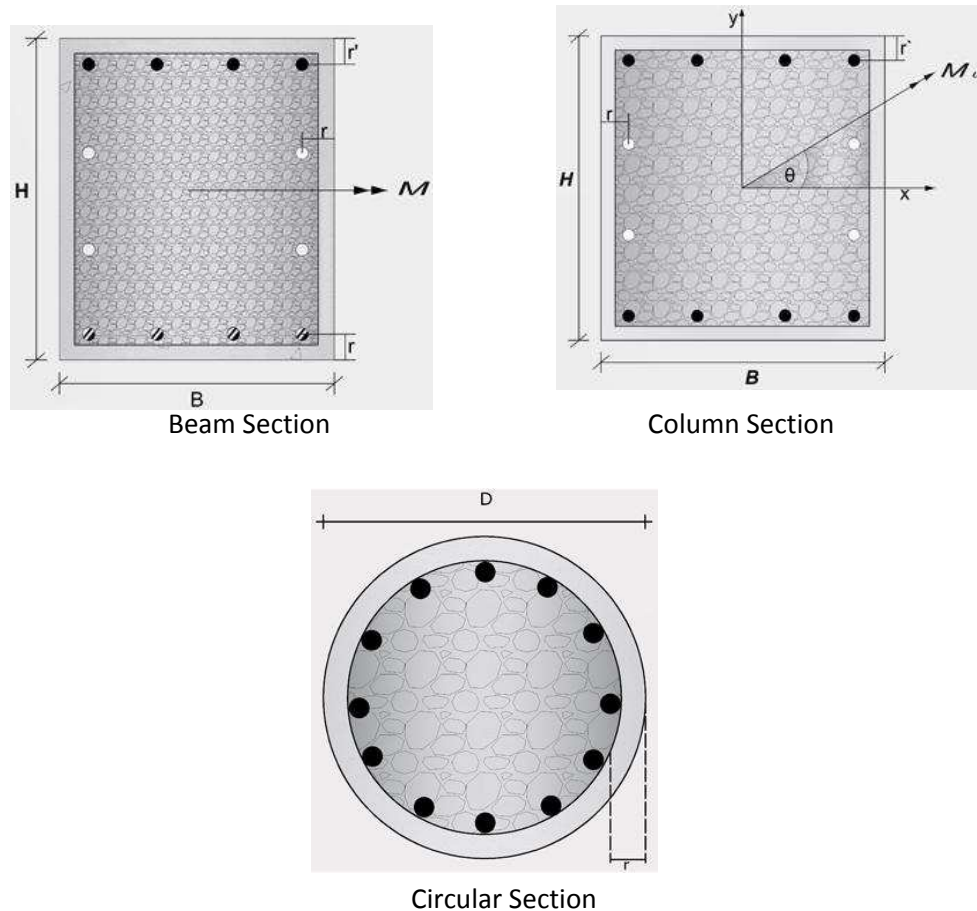


Figure 2 Sections supported in RC-Analysis

RC-Analysis models the nonlinear behavior of confined concrete, unconfined concrete and the reinforcing bars, to obtain the Moment-Curvature response. The shear strength plot is obtained using the modified UCSD model (Kowalsky and Priestley, 2000). From the M-C plot different response limit states can be identified. The M-C and the shear strength response are coupled to show whether moment or shear will control failure of the section. Details of the models and theory implemented in this program are presented in the following section.

HOW IT WORKS

Once the user enters the geometry, reinforcing, material properties and other design parameters, the analysis is performed as follows:

Step 1. Material Models

The default models for the unconfined and confined concrete are those proposed by Mander, Priestley and Park (1988). The default model for steel is the one used by King (1986) in his section analysis program. A detailed description of the models is presented next:

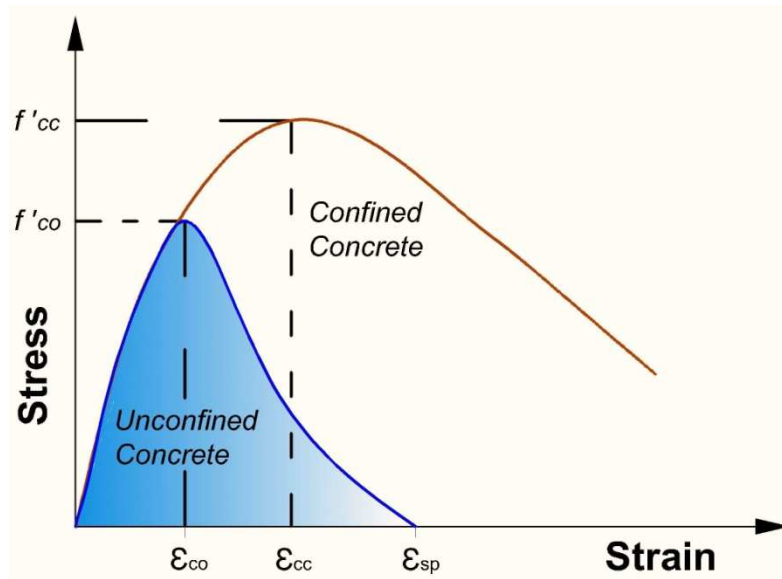


Figure 3 Mander model for confined and unconfined concrete

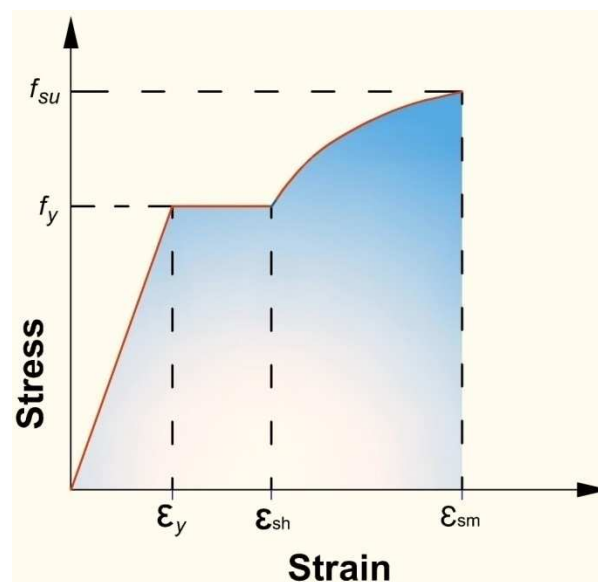


Figure 4 Model for reinforcing steel

Model for the confined concrete

Mander et al. (1998) have proposed a unified stress-strain approach for confined concrete (Fig. 3) applicable to circular and rectangular sections. Since experimental results have shown that the ultimate concrete strain calculated based on the Mander model to be consistently conservative by 50% (Kowalsky, 2000), the original Mander expression for ϵ_{cu} is modified as shown in equation 6. The longitudinal compressive stress, f_c , is given by:

$$f_c = \frac{f'_{cc} x r}{r-1+x^r} \quad (1)$$

Where:

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}} \quad (2)$$

ε_c : Longitudinal compressive concrete strain

$$\varepsilon_{cc} = \varepsilon_{co} \left[1 + 5 \left(\frac{f'_{cc}}{f'_{co}} - 1 \right) \right] \quad (3)$$

ε_{co} : Unconfined concrete strain

f'_{co} : Unconfined concrete stress

$$r = \frac{E_c}{E_c - E_{sec}} \quad (4)$$

$$E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}} \quad (5)$$

$$\varepsilon_{cu} = 1.4 \left(0.004 + \frac{1.4 \rho_s f_{yh} \varepsilon_{su}}{f_{cc}} \right) \quad (6)$$

For circular sections: 1

$$f'_{cc} = f'_{co} \left(-1.254 + 2.254 \sqrt{1 + \frac{7.94 f'_l}{f'_{co}} - \frac{2 f'_l}{f'_{co}}} \right) \quad (7)$$

$$f'_l = \frac{1}{2} K_e \rho_s f_{yh} \quad (8)$$

f_{yh} : Yielding stress of transverse steel

$$\rho_s = \frac{4 A_{sp}}{d_s S} \quad (9)$$

A_{sp} : Cross section area of spiral or hoop

d_s : Diameter of the core (center to center of spirals)

$$k_e = \frac{\left(1 - \frac{s'}{2d_s} \right)^2}{1 - \rho_{cc}} \text{ For circular hoops} \quad (10)$$

$$k_e = \frac{1 - \frac{s'}{2d_s}}{1 - \rho_{cc}} \text{ for circular spirals} \quad (11)$$

s' : Clear distance between spirals or hoops

ρ_{cc} : Ratio of area longitudinal reinforcement to area of core section

For rectangular sections

$$f'_{lx} = k_e \rho_x f_{yh}, f'_{ly} = k_e \rho_y f_{yh} \quad (12)$$

$$\rho_x = \frac{A_{sx}}{sH_c}, \rho_y = \frac{A_{sy}}{sB_c} \quad (13)$$

A_{sx}, A_{sy} : Total area of transverse steel running in the x and y directions, respectively

B_c, H_c : Core dimensions to centerline of perimeter hoop in the x and y directions

s : Distance between hoops, center to center

$$k_e = \frac{\left(1 - \sum_{i=1}^n \frac{(w_i)^2}{6B_c H_c}\right) \left(1 - \frac{s'}{2B_c}\right) \left(1 - \frac{s'}{2H_c}\right)}{(1 - \rho_{cc})} \quad (14)$$

$w_i; i^{th}$: Clear distance between longitudinal adjacent bars

Model for the Unconfined Concrete

The Unconfined Concrete follow the same curve that the confined concrete (eq. 1) with $f'_{cc} = f'_{co}$. The part of the falling branch for strains larger than ϵ_y is assumed to be a straight line which reaches zero at the spalling strain (typically 0.0064).

Model for Reinforcing Steel

The stress-strain relation for the reinforcing steel (fig. 4) is the same used by the King program (1986)

$$\begin{aligned} f_s &= E_s \epsilon_s & \epsilon_s &\leq \epsilon_y \\ f_s &= f_y & \epsilon_y &< \epsilon_s < \epsilon_{sh} \\ f_s &= f_y \left[\frac{m(\epsilon_s - \epsilon_{sh}) + 2}{60(\epsilon_s - \epsilon_{sh}) + 2} + \frac{(\epsilon_s - \epsilon_{sh})(60 - m)}{2(30r + 1)^2} \right] & \epsilon_{sh} &< \epsilon_s \leq \epsilon_{sm} \end{aligned} \quad (15)$$

Where:

$$m = \frac{(f_{su}/f_y)(30r + 1) - 60r - 1}{15r^2} \quad (16)$$

$$r = \epsilon_{su} - \epsilon_{sh} \quad (17)$$

Step 2. Section Mapping

The section is divided in square fibers using a grid as shown in Fig. 5. Depending on their position, each fiber is assigned with the stress-strain model of confined or unconfined concrete. Steel fibers are also created at the location of the longitudinal reinforcing bars and are given the stress strain model of the reinforcing steel.

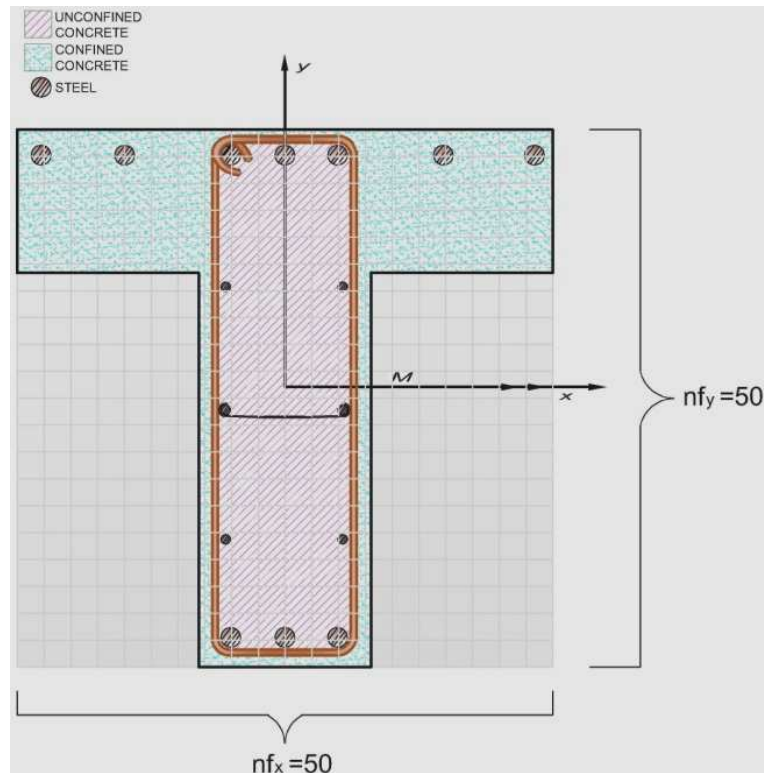


Figure 5 Division of the section

The accuracy of the analysis depends on the number of fibers in which the section is mapped. However, using large numbers of fibers increases the computation time. RC-Analysis uses by default grids of 50x50 fibers for circular and rectangular columns and grids of 1x50 fibers for rectangular beams.

Step 3. Coordinate rotation

When the moment is applied at an angle θ with respect to the section axes (X,Y), the coordinates of each fiber must be transformed to a new set of axes (X',Y'), where X' is in the direction of the applied moment (Fig. 6). The transformation is accomplished with equation (18), and equation (19).

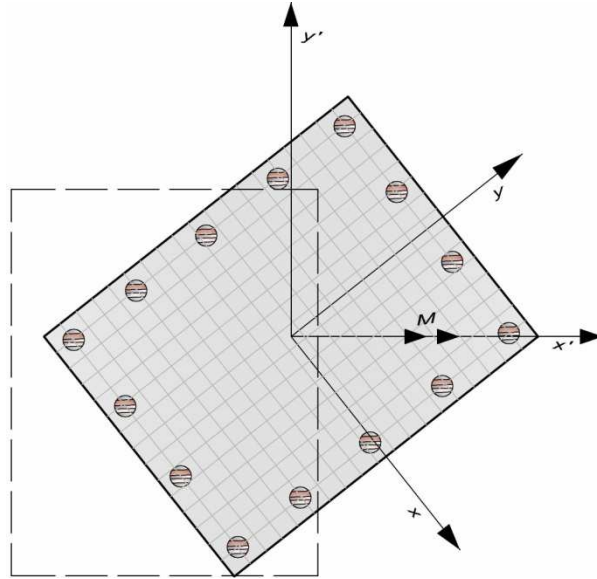


Figure 6 Direction of the applied moment

$$x' = x \cos \theta - y \sin \theta \quad (18)$$

$$y' = x \sin \theta + y \cos \theta \quad (19)$$

Step 4. M-C Analysis

To start the M-C analysis the following parameters are defined:

- a) The number of analysis points. RC-Analysis runs 50 points by default
- b) The concrete strain, ϵ_c , for each analysis point. RC-Analysis varies ϵ_c linearly from 0 to ϵ_{cu}
- c) The neutral axis depth must be guessed

Next, section analysis is performed at each concrete strain level to get a series of M-C points. Section analysis is accomplished as follows:

Step 4.1 Strain computation

For the selected value of ϵ_c and the assumed neutral axis depth c , the curvature in the section is given by Equation (20) then; the strain at each fiber location is computed with Equation (21). This assumes plain sections remain plain after flexure.

$$\phi = \frac{\epsilon_c}{c} \quad (20)$$

$$\varepsilon = \phi(y' - c) \quad (21)$$

Step 4.2. Stress computation

Stress is computed for each fiber on the basis of computed strain and the corresponding stress-strain model.

Step 4.3. Force equilibrium check

The force in each fiber results from the product of the stress in the fiber and the area of the fiber. The area of the concrete fibers is given by the size of the grid. The area of the steel fibers is based on the diameter of the bars.

The internal forces in the section must be in equilibrium with the applied axial load. This is checked using Equation (22), where EQR is the equilibrium error; F is the sum of all fiber forces (compression forces are positive and tension forces are negative) and P is the axial load P (positive).

$$EQR = F - P \quad (22)$$

If EQR is positive, the assumed c is too large and the process is repeated from Step 4.1 with a reduced value of c. If EQR is negative, the assumed c is too small and the process is repeated from Step 4.1 with a larger c. If the absolute value of EQR is smaller than the analysis tolerance (In RC-Analysis Abs (EQR) < 0.0001), analysis is continued.

Step 4.4. Moment computation

Once c has converged, the moment in the section is computed with Equation (23)

$$M = \sum F_i y' \quad (23)$$

Step 4.5. Output

The following data is recorded: Curvature, Moment, Neutral Axis Depth, Concrete Strain, and Steel Strain. Then the analysis is repeated from Step 4.1 with a new value of concrete strain.

Bilinear M-C Diagram

Moment-curvature relations can be well fitted with a bilinear diagram. This is commonly done to ease the calculations based on the moment curvature relation. There are several methods that have been proposed for this task. RC-Analysis produces the bilinear diagram by having a line starting at the origin and passing by the point of first yield, and other line that starts in the point of failure and that cuts the first line in a point where moment equals the nominal moment of the section. The point at which the two lines intercept sets the yield curvature and yield moment of the section. The slope of the first line gives the cracked flexural stiffness, EI , of the section. The yield curvature is an important inherent property of the section that is used in the computation of yield displacement and damage indexes such as ductility.

Shear Capacity

The shear strength envelope for the member is calculated using the revised UCSD shear model (Kowalsky and Priestley, 2000). The original UCSD model was the first model for assessment of shear strength that included: (1) the effect of the axial load separate from the concrete strength and (2) the degrading of concrete strength with ductility. The revised model intends to take also into account: (1) the effect of concrete compression zone on the mobilization of transverse steel and (2) the influence of the aspect ratio and the longitudinal steel ratio in the shear strength of the concrete. The model expresses the shear strength capacity of the member as the sum of three separate components as shown in equation 24. V_s represent the shear Capacity attributed to the steel truss mechanisms, V_p represents the strength attributed to the axial load and V_c represents the strength of the concrete shear resisting mechanism.

$$V = V_s + V_p + V_c \quad (24)$$

$$V_s = \frac{\pi}{2} A_{sp} f_{yh} \frac{D-clb+\frac{d_h}{2}-c}{s} \cot(\theta) \text{ Circular section} \quad (25)$$

$$V_s = A_{sx} f_{yh} \frac{H-clb+\frac{d_h}{2}-c}{s} \cot(\theta) \text{ Rectangular section} \quad (26)$$

Where clb is the cover to the longitudinal bar, d_h is the diameter of the transverse steel and c is the depth of the neutral axis at M_n . The concrete contribution is obtained with the following equation, where the coefficients $\alpha \beta \gamma$ are given in Fig. 7.

$$V_c = \alpha \beta \gamma \sqrt{f'_c} (0.8 A_g) \quad (27)$$

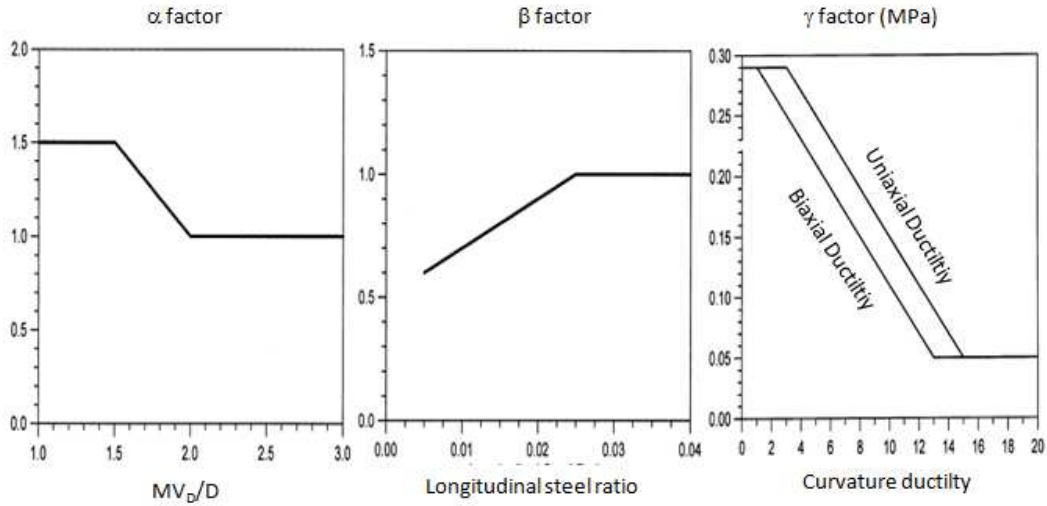


Figure 7 α β γ factors of UCSD modified model

$$V_p = P \frac{D-c}{2L} \quad P > 0 \quad \text{Single bending} \quad (28)$$

$$V_p = P \frac{D-c}{L} \quad P > 0 \quad \text{Double bending} \quad (29)$$

$$V_p = 0 \quad P < 0 \quad (30)$$

Equations 24 to 30 are used for the assessment of the shear strength of existing structures. For the design of new structures a more conservative approach is used: the axial load is reduced by 15%, the angle of the flexure-shear crack is incremented to 35° and a shear strength reduction Factor of 0.85 is applied.

VERIFICATION EXAMPLES

Example1. Analysis of a circular column

A 1000 mm diameter RC column section is analyzed to obtain its moment-curvature response. The concrete in the column has a compression strength $f'_c = 24$ MPa and it is reinforced to resist flexure with 24D25 bars. The shear reinforcement in the section is given by a 12mm spiral spaced 150mm. Since the column is in double bending, the shear span in one half of its height, which results in $H_s = 3$ m. The cover to the centre of the main reinforcement is 50 mm. The yield strength of the reinforcing bars and spiral is $f_y = 420$ MPa, with a strain hardening ratio of 1.4. The axial load acting on the column is 3000 kN. The section and material properties as entered in RC-Analysis are shown in Fig. 8



VIRTUAL LABORATORY OF EARTHQUAKE ENGINEERING

RC ANALYSIS : CIRCULAR COLUMN MOMENT-CURVATURE AND SHEAR CAPACITY OF RC SECTIONS

SECTION PROPERTIES

| | |
|---|---|
| <input type="text" value="1000"/> | SECTION DIAMETER D (mm) |
| <input type="text" value="50"/> | COVER r (mm) |
| <input type="text" value="25"/> | LONGITUDINAL REBAR DIAMETER d_{bl} (mm) |
| <input type="text" value="24"/> | # LONGITUDINAL REBARS |
| <input type="text" value="12"/> | TRANSVERSE REBAR DIAMETER d_{bt} (mm) |
| Spiral <input type="button" value="v"/> | TRANSVERSE REINFORCEMENT TYPE |
| <input type="text" value="150"/> | SPACING OF TRANSVERSE REBAR s_t (mm) |
| <input type="text" value="3000"/> | AXIAL LOAD P (KN) |
| <input type="text" value="3000"/> | SHEAR SPAN (mm) |
| uniaxi <input type="button" value="v"/> | DUCTILITY MODE |

MATERIAL PROPERTIES

| | |
|----------------------------------|--|
| <input type="text" value="24"/> | UNCONFINED COMPRESSION CONCRETE STRENGTH f_c (Mpa) |
| <input type="text" value="420"/> | YIELD STRESS OF LONGITUDINAL REBAR f_y (Mpa) |
| <input type="text" value="420"/> | YIELD STRESS OF TRANSVERSE REBAR f_{yh} (Mpa) |
| <input type="text" value="1.4"/> | STEEL HARDENING RATIO (1.20-1.40) |

SECTION

CONCRETE MODEL

STEEL MODEL

Figure 8 Example data as entered in RC-Analysis

RESULTS

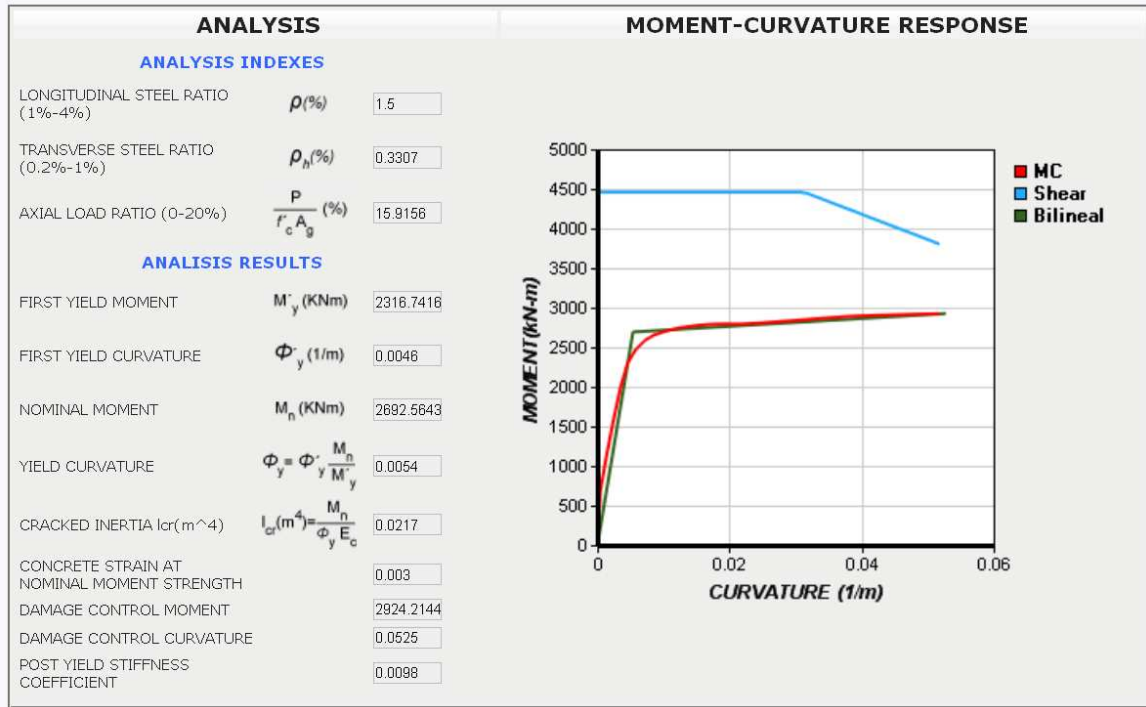


Figure 9 Main analysis results

The analysis results in the graph and data shown in Fig. 9. For verification purposes the same section is analyzed with Response 2000 (Bentz, 2000), the program USRC (Esmaily, 2001) and CUMBIA (Montejo, 2007) and resulting M-C Diagrams are shown in Fig. 10. Along with the one obtained with RC-Analysis.

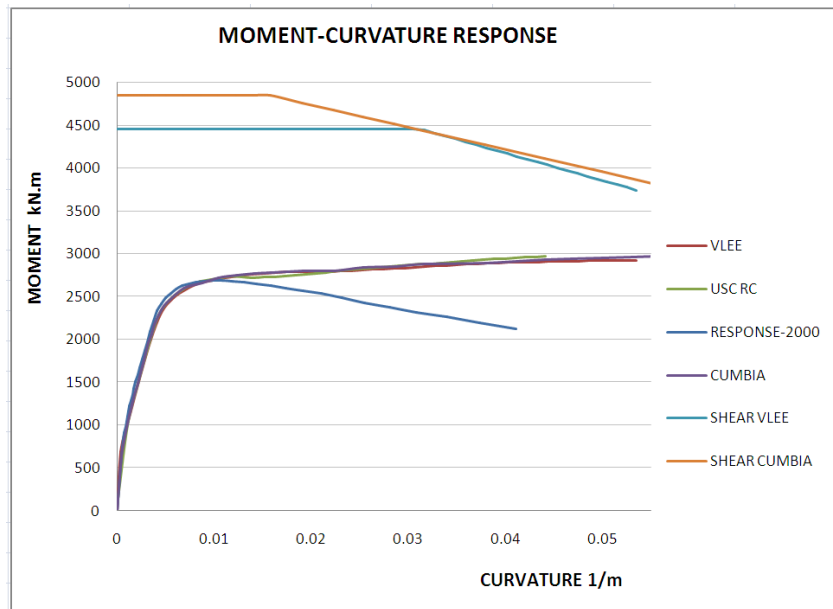


Figure 10 Comparison of M-C curves

From the analysis of Fig. 10 it is concluded that RC-Analysis produces a M-C curve that is very close to other curves found with programs such as Cumbia and USC RC that use Mander's models for concrete. The shear capacity curve is also close to that found with Cumbia.

References

Bentz E. RESPOSE-2000 "Load-Deformation Response of Reinforced Concrete Sections". Software, Version 1.1 September 2001.

Esmaily A. USC-RC "Software for analyzing behavior of a single reinforced concrete member", Version 1.0.2, USC Civil Engineering Department, 2001.

King D.J., Priestley M.J.N., and Park R. Computer Programs for Concrete Column Design. Research Report 86/12, Department of Civil Engineering, University of Canterbury, New Zealand, May 1986.

Kowalsky M.J. and Priestley M.J.N. Improved Analytical Model for Shear Strength of Circular Reinforced Concrete Columns in Seismic Regions. ACI Structural Journal, Vol. 97, No. 3, 2000.

Mander J.B., Priestley M.J.N., and Park R. Theoretical Stress-Strain Model for Confined Concrete. ASCE Journal of Structural Engineering, Vol. 114, No. 8, August 1988.

Montejo L. CUMBIA "Generation Set of codes for the analysis of reinforced concrete members". Technical Report No. IS-07-01, February 2007.